

Sheet (5)

5.16. (a) Verify the time-shifting property (5.50), that is,

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$$

5.17. Verify the frequency-shifting property (5.51), that is,

$$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$$

5.18. Verify the duality property (5.54), that is,

$$X(t) \leftrightarrow 2\pi x(-\omega)$$

5.19. Find the Fourier transform of the rectangular pulse signal $x(t)$ [Fig. 5-16(a)] defined by

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases} \quad (5.135)$$

5.20. Find the Fourier transform of the signal [Fig. 5-17(a)]

$$x(t) = \frac{\sin at}{\pi t}$$

5.21. Find the Fourier transform of the signal [Fig. 5-18(a)]

$$x(t) = e^{-a|t|} \quad a > 0$$

Signal $x(t)$ can be rewritten as

$$x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$$

5.23. Find the Fourier transforms of the following signals:

- | | |
|--------------------------------------|-------------------------------------|
| <i>(a)</i> $x(t) = 1$ | <i>(b)</i> $x(t) = e^{j\omega_0 t}$ |
| <i>(c)</i> $x(t) = e^{-j\omega_0 t}$ | <i>(d)</i> $x(t) = \cos \omega_0 t$ |
| <i>(e)</i> $x(t) = \sin \omega_0 t$ | |

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5.24. Find the Fourier transform of a periodic signal $x(t)$ with period T_0 .

We express $x(t)$ as

5.25. Find the Fourier transform of the periodic impulse train [Fig. 5-22(a)]

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

5.26. Show that

$$x(t) \cos \omega_0 t \leftrightarrow \frac{1}{2}X(\omega - \omega_0) + \frac{1}{2}X(\omega + \omega_0) \quad (5.148)$$

and $x(t) \sin \omega_0 t \leftrightarrow -j \left[\frac{1}{2}X(\omega - \omega_0) - \frac{1}{2}X(\omega + \omega_0) \right] \quad (5.149)$

5.28. Verify the differentiation property (5.55), that is,

$$\frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega)$$

5.45. Consider a continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \quad (5.166)$$

Using the Fourier transform, find the output $y(t)$ to each of the following input signals:

$$(a) \quad x(t) = e^{-t}u(t)$$

$$(b) \quad x(t) = u(t)$$